$$\zeta(s)=s\left(\frac{1}{s-1}-\int_1^{\infty}\frac{x-\lfloor x\rfloor}{x^{s+1}}dx\right)\frac{s}{s-1} \qquad \textbf{(1-2)}$$

$$\zeta(s)=\frac{s}{s-1}-\sum_{n=1}^{b-1}\left(s\int_n^{n+1}\frac{x-n}{x^{s+1}}dx\right)-s\int_b^{\infty}\frac{x-\lfloor x\rfloor}{x^{s+1}}dx \qquad \textbf{(2-1)}$$

$$=\frac{s}{s-1}-\sum_{n=1}^{b-1}\left(s\int_n^{n+1}\frac{x-n}{x^{s+1}}dx\right)-\underbrace{s\int_b^{\infty}\frac{x-\lfloor x\rfloor}{x^{s+1}}dx}_{O_1} \qquad \textbf{(2-1-1)}$$

$$=\frac{s}{s-1}-s\int_1^{b}\frac{x}{x^{s+1}}dx-s\sum_{n=1}^{b-1}\left(\int_n^{n+1}\frac{n}{x^{s+1}}dx\right)-O_1 \qquad \textbf{(2-1-2)}$$

$$=\frac{s}{s-1}-s\int_1^{b}\frac{1}{x^{s}}dx-s\sum_{n=1}^{b-1}n\left(\int_n^{n+1}\frac{1}{x^{s+1}}dx\right)-O_1 \qquad \textbf{(2-1-3)}$$

$$=\frac{s}{s-1}-\frac{s}{1-s}b^{1-s}-\frac{s}{s-1}-s\sum_{n=1}^{b-1}n\left(\int_n^{n+1}\frac{1}{x^{s+1}}dx\right)-O_1 \qquad \textbf{(2-1-4)}$$

$$=\left(1-\frac{1}{1-s}\right)b^{1-s}-s\sum_{n=1}^{b-1}n\left(\int_n^{n+1}\frac{1}{x^{s+1}}dx\right)-O_1 \qquad \textbf{(2-1-5)}$$

$$=\frac{b}{b^s}-\frac{b^{1-s}}{1-s}-s\sum_{n=1}^{b-1}n\left(\int_n^{n+1}\frac{1}{x^{s+1}}dx\right)-O_1=\frac{b}{b^s}-\frac{b^{1-s}}{1-s}-s\sum_{n=1}^{b-1}\frac{1}{s}\left(\frac{n}{(n+1)^s}-\frac{n}{n^s}\right)-O_1 \qquad \textbf{(2-1-6)}$$

$$=\frac{b}{b^s}-\frac{b^{1-s}}{1-s}-\frac{s}{s}\left(\frac{1}{2^s}-\frac{1}{1^s}+\frac{2}{3^s}-\frac{2}{2^s}+\cdots+\frac{b-1}{(b-1+1)^s}-\frac{b-1}{(b-1)^s}\right)-O_1 \qquad \textbf{(2-1-7)}$$

$$=\frac{b}{b^s}-\frac{b^{1-s}}{1-s}+\left(\frac{1}{1^s}-\frac{1}{2^s}+\frac{2}{2^s}-\frac{2}{3^s}+\cdots+\frac{b-1}{(b-1)^s}-\frac{b-1}{b^s}\right)-O_1 \qquad \textbf{(2-1-8)}$$

$$=\frac{b}{b^s}-\frac{b^{1-s}}{1-s}+\left(\sum_{n=1}^{b-1}\left(\frac{1}{n^s}\right)-\frac{b}{b^s}+\frac{1}{b^s}\right)-O_1=\sum_{n=1}^{b}\left(\frac{1}{n^s}\right)-\frac{b^{1-s}}{1-s}-s\int_b^{\infty}\frac{x-\lfloor x\rfloor}{x^{s+1}}dx\,,b\in\mathbb{N} \qquad \textbf{(2-1-9)}$$